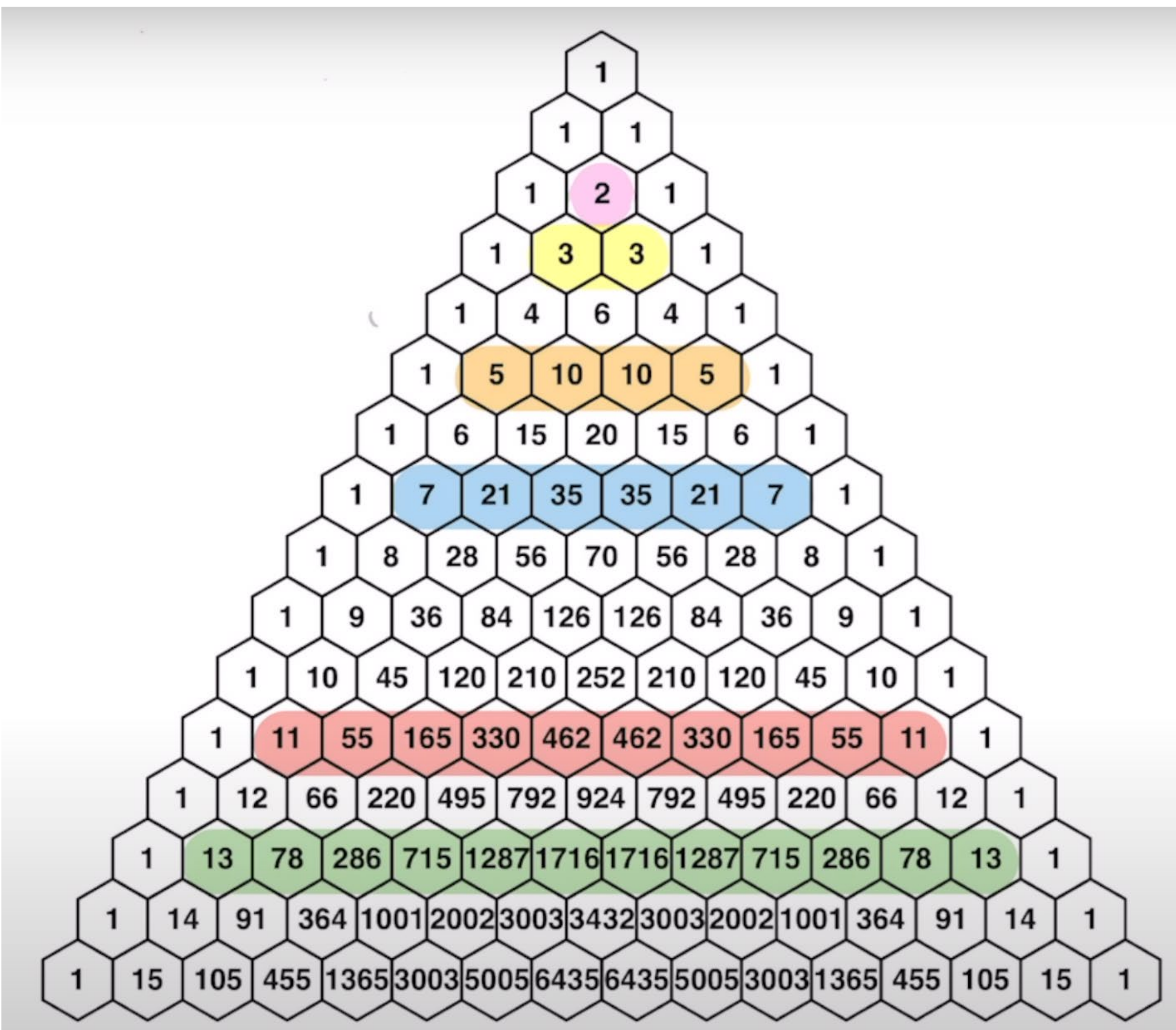
Pascal's Triangle, Prime Numbers, and… Count Dracula?

Here is a fun little IFA GB math story that I stumbled into with a surprising and sinister conclusion.

I was looking at the first few prime numbers in Pascal's Triangle (PT): 2,3,5,7,11, and 13. I define a horizontal line in PT to be a **prime number line** when the second entry is a prime number. These 6 prime number lines in PT are highlighted in color below:



You will notice that every entry on a prime number line is a multiple of the prime number except for the end 1's. For example, the entries in light blue row 7 (1 7 21 35 35 21 7 1) are all divisible by 7 except the 1's. **The multiplicities of the second entry only occur in PT for prime number lines.**

Could this observation about PT prime number lines offer anything *useful* for understanding the IFA GB? Let's see.

---------------------- BOTTOM PT LINE ON THE IFA GB --------------

The light green line for prime number line 13 in the above PT is the PT line at the bottom the IFA GB:

Writing this prime number line gives

1 **13** 78 286 715 1287 1716 1716 1287 715 286 78 **13** 1 (1)

with a total of 2**13 = 8192. These 14 entries are the number of different paths a bead can take to reach the top of each of the 14 pegs in the bottom row of the IFA GB. The probability of a bead hitting the ith peg is the ith (0 to 13) number divided by 8192.

------------------ NORMALIZED EQN. (1) FOR THE IFA GB -------------------

The number 13 is a prime number. Therefore, the PT entries in Eqn. (1) are all multiples of 13 except the 1's. Dividing the 14 entries in Eqn. (1) by 13 and defining 1/13 to be zero, the normalized form of PT Eqn. (1) becomes

0 1 6 22 55 99 132 132 99 55 22 6 1 0 (2)

with a total of 630. All the normalized PT numbers in Eqn. (2) are whole numbers because of the multiplicity property of the entries in a PT prime number line. These normalized numbers are obviously no longer than the total paths a bead can take to get to a bin. The smaller numbers and the resulting peg probabilities in Eqn. (2) are easier to visualize when compared to Eqn. (1). The first and last entries of Eqn. (2) are now 0 compared to a 1 in Eqn. (1), which seems acceptable since the probability of a bead being at either end position is very small (1/8192 = 0.00012).

----------- BEAD-BIN PT LINE USING THE NORMALIZED EQN. (2) --------

Creating the next normalized line in PT using Eqn. (2) and the PT rule of adding the two numbers above each of the 15 locations gives the following line of whole numbers:

0 1 7 28 77 154 231 264 231 154 77 28 7 1 0 (3)

with a total of 1260. The probability of a bead landing in one of the 15 bins is the ith entry divided by 1260. For example, the probability of a bead landing in the center bead bin 7 using Eqn. (3) is 264/1260 = 0.20952.

To compare normalized Eqn. (3) with the equivalent unnormalized bead-bin line in PT, here is the unnormalized PT line after the prime number line in Eqn. (1):

1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1 (4)

with a total of 2**14 = 16384. If 16384 beads were used in the GB, the entries in the above line would be the expected number of beads in each of the 15 bead bins. The probability of a bead landing the center bead bin 7 is 3432/16384 = 0.20947. The slight difference compared to the Eqn. (2) probability is because 1/13 was set to zero in Eqn. (3).

----- USING EQN. (3) TO SELECT THE NUMBER OF BEADS N IN A GB ------

The total bead-bin number 1260 suggests that multiples of 1260 would be a good choice for the total beads because then multiples of the normalized Eqn. (3) entries could be used for the number of expected beads.

What multiple should be used? Two selection criteria quickly come to mind: bin 2 (and 13) should have at least 30 beads and bin 3 (and bin 12) should have at least 100 beads. 30 is a minimum accepted adequate sample size and 100 is a very good sample size. The bin 2 criteria is the more stringent. It requires a multiplicative factor of 5 since 5*7 = 35, which is greater than 30. Therefore, 5*1260 = 6300 beads would be the desired number to meet this criterion. Your 6000 beads is very nearly this value.

--------- HOW DOES COUNT DRACULA BECOME INVOLVED HERE? --------------

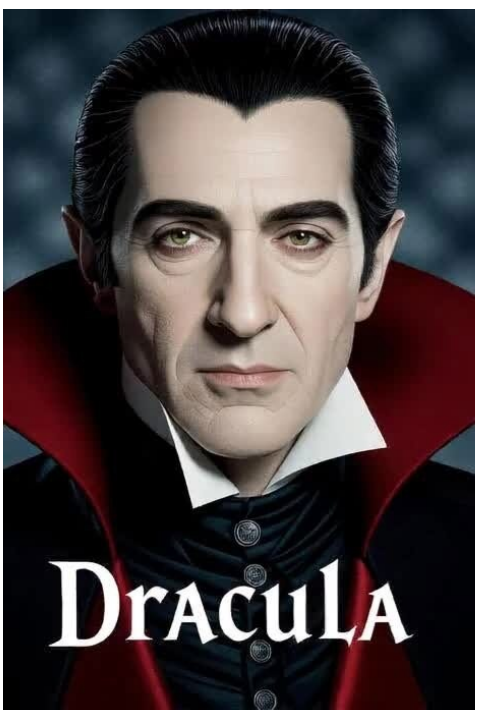
So how does Count Dracula become involved in this analysis? Using the normalized PT 13 prime number line [Eqn. (2)] to define the normalized PT bead-bin probability line [Eqn. (3)] produces 15 entries totaling 1260. It turns out that 1260 is a very special number. **1260 is the first Vampire Number!**

Quoting from https://en.wikipedia.org/wiki/Vampire_number,

*A **vampire number** (or **true vampire number**) is a composite natural number with an even number of digits, that can be factored into two natural numbers each with half as many digits as the original number, where the two factors* *contain precisely all the digits of the original number, in any order, counting multiplicity. The two factors cannot both have trailing zeroes. The first vampire number is 1260 = 21 × 60.*

*NOTE: These two factored numbers are called the **fangs** of the composite number. Vampire numbers can have more than one pair of fangs. Vampire numbers were first defined in 1994 by Clifford A. Pickover to describe numbers that are "subtly hidden" from our view, just as vampires walk amongst mere mortals in science fiction novels. There are 7 4-digit Vampire Numbers: 1290, 1395, 1435, 1530, 1827, 2187, and 6880.*

So, by this rigorous (😳) PT prime number analysis, Count Dracula and his friends exist in the bead bins of the IFA GB.



You could easily turn your IFA GB into a Vampire Pro Model by using 6880 beads. 😳

---- OTHER IDEAS ABOUT PT PRIME NUMBER LINES AND THE IFA GB? -----

I hope you enjoyed this fun little exercise with PT prime line numbers. What do you think? Do you have any other ideas about how to use the unique multiplicity property of the PT 13 prime number line for anything fun or even "useful" related to the IFA GB?

You might want to lock up your GBs at night just to be safe.

PS What are Vampire Numbers Used For?

Vampire numbers are not directly used in real-world applications, but they are often used in educational contexts, such as in programming languages, to test the skills of students. They serve as a challenge for those learning to write efficient code that can find specific patterns or structures within numbers. The process of finding vampire numbers is a common exercise in computer science and programming education, helping to reinforce concepts of number manipulation and pattern recognition.

PPS Here is a Video About Vampire Numbers

[Vampire Numbers - Numberphile]

[Vampire Numbers - Numberphile]

To mark Halloween we're discussing vampire numbers. More links & stuff in full description below ↓↓↓ More cool sciencey Halloween videos at http://www.youtube.com/playlist?list=PL9eEsN9D48mf8CIYU1wRnWak0lcAnJN1u This video features Dr James Grime and Professor Ed Copeland. NUMBERPHILE Website: http://www.numberphile.com/ Numberphile on ...

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