Can You Do Well While Doing Good?

Investors have long been rewarded for the capital they provide to the global markets. For some, investing is purely about return. For others, attitudinal preferences play an important role in investment decisions. Catholics, in particular, are a great example of this idea. Their preferences can carry significance that goes beyond the realm of simple risk and return analysis. And, because consumer preferences ultimately affect the success or failure of any given business or industry, a group of like-minded individuals can influence future generations based on their investment actions.

Catholic institutions have long been concerned with returns. Thus, the question inevitably arises: “Will investing according to the Catholic investing guidelines hamper expected returns?”

Mark Hebner, Mary Brunson and Nobel Prize winner Harry Markowitz dig into this question, providing findings that are pivotal to the decision-making process for Catholic institutional investment committees who seek to understand their ability to “do well while doing good.”
This is the second of two essays which consider how much is lost by confining investments to a socially responsible universe. In other words, can you do well for your investors while doing good for the world? The first of these two essays (by Mary Brunson) is empirical, and shows that in fact little is lost by thus restricting the universe of permitted securities. The present essay is theoretical. In it I explain why I am not a bit surprised by the empirical conclusion.

To understand Modern Portfolio Theory (MPT) as it relates to the present question, you need to understand the four concepts listed in Exhibit 1; namely, Standard Deviation, Variance, Correlation and Covariance.

The standard deviation of a probability distribution describes how spread out the distribution is. If you sample repeatedly from the same distribution, the standard deviation is a measure of how volatile the time series is that you generate. In fact, sometimes the standard deviation of a return series is referred to as its “volatility.” Most of a probability distribution lies between its average (or “mean”) value minus two standard values and the average plus two standard deviations. The exact probability of being in this range depends on the shape of the distribution. But it is a mathematical truth that most of a probability distribution—at least 75 percent and usually around 90 or 95 percent of it—lies between the average plus or minus two standard deviations (AVG ± 2 StdDev).

Variance is the standard deviation-squared. It has no intuitive significance. In particular, it is not true that most of a probability distribution lies between the average plus or minus two variances (as distinguished from plus or minus two standard deviations). The only reason that variance is used so prominently in modern portfolio theory is that formulas are much simpler in terms of variance. We first compute variance, and then take its square root to find standard deviation.

The correlation coefficient (or just “correlation” for short) is a number between -1.0 and +1.0. It measures the extent to which two series, or two random variables, move up and down together. If two series move up and down together along a positively sloped straight line, with no deviation from this line, then they have a correlation of 1.0. Conversely, if one always moves down when the other moves up, and vice versa, along a negatively sloped straight line, then their correlation is -1.0. If information about one provides no information (on average) about the other, their correlation is 0.0, and they are said to be uncorrelated. When correlations among stock returns are measured over moderately long periods they usually have values in the order of 0.25, but this varies from decade to decade as well as from stock to stock.

Like variance, covariance defies intuitive explanation. It too plays a prominent part in MPT because of its usefulness in computation, and because of the crucial role that covariances among securities play in determining the
volatility of large portfolios. As noted in Exhibit 1, the covariance between two series (or two random variables) equals their correlation times the standard deviation of one of them times the standard deviation of the other.

The law of the average covariance.
The formula for the variance of a portfolio depends on the amounts of the various securities held in it and their respective variances and covariances. The formula is somewhat complicated, and is to be found in all (that I know of) finance textbooks. The formula simplifies considerably if we consider the special case of an “equal weighted portfolio,” that is, when equal dollar amounts are invested in each security. In this case the variance of the portfolio obeys the “law of the average covariance” as displayed in Exhibit 2. This says that portfolio variance is the weighted average of the two parts: the average of security variances and the average of the covariances among pairs of securities. The first term—the average variance—is computed by adding up all security variances and dividing by the number of these. The second term, the average covariance, is computed by adding up all covariances, i.e., the covariances between each security and each other security—and dividing by the number of such pairs of securities.

These two quantities are multiplied by weights and added together. For example, if there are $N = 100$ securities in an equally weighted portfolio, then the variance of the portfolio equals $.01 (=1/\ N)$ times the average of security variances, plus $.99 (=1-1/\ N)$ times the average of all covariances.

Clearly, as $N$ increases the variance of portfolio approaches the average covariance, as noted in Exhibit 3. The latter exhibit shows us what portfolio variance approaches as $N$ grows large. Exhibit 2 shows us how fast it gets there. As I will illustrate shortly, Exhibit 3 tells us that diversification has a limited ability to reduce portfolio volatility in the face of correlated risks. Therefore, in order to reduce portfolio volatility to a comfortable level, most investors (including institutional investors) must combine their equity portfolio with a fixed income portfolio. Exhibit 2, on the other hand, tells us that it does not require hundreds of securities to achieve most of the benefits of diversification.

Before I illustrate Exhibits 2 and 3 numerically, let me emphasize that these are mathematical relationships rather than empirical relationships. They will not be refuted by some statistical analysis next year. Of course, to estimate the quantity on the left in Exhibit 3 one must estimate the quantity on the right. But at least the exhibit makes clear what needs to be estimated.

Numerical examples
To illustrate the messages in Exhibits 2 and 3, let us consider cases in which all securities are equally volatile and all pairs of securities are equally correlated. First consider Exhibit 3 in this case. If we combine the fact, displayed in Exhibit 3 (that the variance of the portfolio approaches the average covariance) with the formula for covariance in Exhibit 1, we find in the present case that—as displayed in Exhibit 4—the variance of the portfolio approaches the security variance times the correlation. But recall that the volatility of a portfolio is its standard deviation rather than its variance. Taking the square root of the two
sides of the first equation in Exhibit 4, we derive the second equation: as the number of securities in an equally weighted portfolio increases, the volatility (standard deviation) of the portfolio approaches the volatility of securities times the \textit{square root} of the correlation coefficient.

Table 1 illustrates this relationship numerically. If each pair of securities had a (not atypical) correlation of 0.25 the volatility of the portfolio would be 0.50 times the volatility of each security. \emph{Think about that!} If all pairs of securities had a 0.25 correlation, then the volatility of the portfolio would be at least half as great as that of a single security! If correlations were as low as 0.1 the volatility of the portfolio would be no lower than about 32 percent of that of a completely undiversified portfolio. Diversification does reduce risk, but it has a severely limited ability to do so in the face of correlated risks. In particular, one should not expect a highly diversified all-equity portfolio to have low volatility. One must add high-grade not-too-long-term fixed income securities to achieve that.

The last line of Table 1 shows that the formulas in Exhibit 4 imply that portfolio volatility could be driven as close to zero as you want—with sufficient diversification—if returns were uncorrelated. But they are not.

Next let us consider how fast volatility approaches the lower bound in Exhibit 3, according to the formula given in Exhibit 2. Table 2 shows this in case of correlation = 0.25. In the table we assume that a very well diversified portfolio has a standard deviation of 20 percent—approximately that of the S&P 500. From this, and the assumed correlation coefficient, we can calculate (from Exhibit 3) that the standard deviation of an individual security must be 40 percent. However, when we square standard deviation to get variance, and then perform the calculation in Exhibit 2, magnitudes are better behaved if we express a 3 percent return rather than \( r = 3.0 \) Then the assumed standard deviation of a security is 0.4 (rather than 40) and its variance is 0.16 (rather than 1600).

The first column of Table 2 is the number of securities in the portfolio; the second column shows portfolio variance for the case under consideration, when a 3 percent return is represented by 0.03 rather than 3.0; the third column is the square root of the second, therefore is the standard deviation of portfolio return when a 3 percent return is written as 0.03; and the final column is portfolio standard deviation when a 3 percent return is represented as \( r = 3.0 \).

The first row of the table shows that if a portfolio has only one security its standard deviation is—of course—equal to the standard deviation of the security. The last line of the table shows that the standard deviation of an equally weighted portfolio with 10,000 securities is practically indistinguishable from that of an “infinitely diversified” portfolio, namely equals “20.00 percent” in the present case.

But note that portfolio standard deviation has fallen from 40 percent to below 21 percent by \( N = 30 \) when there are as few as thirty securities in the portfolio! The relationship between volatility
and N in this example is shown in Figure 1. If the numbers in the table and figure seem implausible, think about the difference between the probability distributions of the average number of heads in a series of coin-flips. If you flip the coin once you will get either one head or one tail: the number of heads will equal 0% or 100%. On the other hand, if you flip it 30 times it is unlikely that you will get 30 heads in a row, or 30 tails in a row. The average number of heads is much more likely to be close to 15, which is the “average” of the number of heads in your “portfolio” of coin tosses. This example uses uncorrelated random variables but, as Exhibit 3 shows, something similar happens with correlated random variables.

The particulars of Table 2 and Figure 1 depend on the particulars of the example, e.g., equal weighted portfolio, all stocks have the same variances, etc. But the formula in Exhibit 2, from which the numbers in Table 2 were computed, is a mathematical law which, therefore, cannot be refuted by an empirical study next year, next decade or next century.

**Caveats and conclusions**

The formula in Exhibits 2 and 3, as illustrated by Tables 1 and 2, show that the ability of diversification to reduce risk is surprisingly limited when returns are correlated; but much of what diversification can achieve is obtained with an even-more-surprisingly few securities. If you would like to see the proofs of the formulae in Exhibits 2 and 3, you will find them in my 1959 book, “Portfolio Selection: Efficient Diversification of Investments.” The proof of the limiting result in Exhibit 3 is in Chapter 5; the result (for any N) in Exhibit 2 is in the Supplement to Chapter 5, in the back of the book, in the second edition. When these results are brought from the classroom into the real world, however, several caveats should be kept in mind.

1. In fact, not all covariances are equal. The formula for covariance in Exhibit 1 implies that if a pair of securities is twice as correlated as the average pair, then their covariance is twice as great. Alternatively, if each has twice the standard deviation then their covariance will be four times as great. Finally, if two securities have twice the correlation and each has twice the standard deviation, then their covariance is eight times as great. Therefore do not expect a huge portfolio consisting solely of many emerging market stocks to have low volatility.

2. Even if every pair of stocks had the same covariance as every other pair of stocks, one would not necessarily want to hold them equally. For one thing, not every stock has the same expected return. Even if all variances and covariances were the same, an investor should tilt its portfolio towards stocks with higher expected return. The extent to which the investor should do so depends on their willingness to take on more risk in exchange for greater return on average. In other words, it depends on where they want to be on the risk-return trade-off curve.

3. The caveat in (2) needs a caveat. The “expected returns” I refer to there should be “forward looking” rather than past averages. But whose forward looking estimates should the investor use? If they can use Warren Buffet’s current...
personal estimates—without paying the price of Berkshire Hathaway stock, which may already fully discount Buffet’s abilities!—then they would probably outperform “the market.” But few money managers are Warren Buffet.

(4) Even passive, “buy and hold,” portfolios should not be equally weighted, because stocks are not equally liquid. Assuming that the two software companies have equally attractive prospects, a large portfolio should have a larger position in Microsoft than in Fly-By-Night Software. Even many, many small portfolios would be ill advised (by some TV personality) to all try to establish substantial positions in a small company. Their similar actions would drive up the stock price of the small company, when they all try to buy, and would drive it down again later when the same or a different TV personality tells them to sell. Nevertheless, relatively small positions in the stocks of smaller companies—commensurate with their market capitalization or trading volume—is generally considered prudent.

The above caveats imply that thirty securities may usually be a little too skimpy for a large portfolio. But, looking at Table 2 again, it seems safe to say that an ethics screen which reduced available securities from about 8,000 to about 4,000 would have to be quite strange to make it impossible to select a reasonably liquid, well diversified portfolio with returns comparable to those usually finds in portfolio of well-established companies with similar levels of portfolio volatility.

The empirical companion piece to this paper reports that, indeed, efficient portfolios from the 4,000+ names of the ethically screened universe lose little in efficiency as compared to those from the full, 8,000+ name universe.
Exhibit 1

Four concepts from MPT

**Standard Deviation**
A measure of “dispersion” or “volatility”.

**Variance** = (Std Dev)²

**Correlation**
A measure of how closely two time series or two random variables move up and down together.

**Covariance between two time series or random variables**
Equals their correlation times the standard deviation of the first times the standard deviation of the second.

Exhibit 2

The variance of an equally weighted portfolio containing N securities is:

\[
\begin{pmatrix}
\text{Portfolio Variance} \\
\end{pmatrix}
= \frac{1}{N}
\begin{pmatrix}
\text{Average Variance} \\
\end{pmatrix}
+ \left(1-\frac{1}{N}\right)\begin{pmatrix}
\text{Average Covariance} \\
\end{pmatrix}
\]
Exhibit 3

As the number of securities in an equally weighted portfolio increases the variance of the portfolio approaches the average covariance:

\[
\text{Portfolio Variance} \rightarrow \text{Average Covariance}
\]

Exhibit 4

When all securities have the same volatility, and all correlation coefficients are the same, Exhibits 1 and 3 imply that, as the number of securities increases:

\[
\left(\frac{\text{Portfolio Variance}}{\text{Variance}}\right) \rightarrow \left(\frac{\text{Correlation Coefficient}}{\text{Coefficient}}\right) \times \left(\frac{\text{Security Variance}}{\text{Variance}}\right)
\]

Therefore

\[
\left(\frac{\text{Portfolio Standard Deviation}}{\text{Standard Deviation}}\right) \rightarrow \sqrt{\frac{\text{Correlation Coefficient}}{\text{Coefficient}}} \left(\frac{\text{Security Standard Deviation}}{\text{Deviation}}\right)
\]
### Table 1

**Portfolio Volatility vs. Security Volatility** For highly diversified portfolios

<table>
<thead>
<tr>
<th>Correlation coefficient</th>
<th>Portfolio volatility as a fraction of security volatility</th>
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</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.50</td>
</tr>
<tr>
<td>0.10</td>
<td>0.32</td>
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<tr>
<td>0.00</td>
<td>0.00</td>
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</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Number of Securities</th>
<th>Portfolio Variance (fraction 1)</th>
<th>Portfolio StdDev (fraction 2)</th>
<th>Portfolio StdDev (percent 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1600</td>
<td>0.4000</td>
<td>40.00</td>
</tr>
<tr>
<td>2</td>
<td>0.1000</td>
<td>0.3162</td>
<td>31.62</td>
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<tr>
<td>3</td>
<td>0.0800</td>
<td>0.2828</td>
<td>28.28</td>
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<tr>
<td>4</td>
<td>0.0700</td>
<td>0.2646</td>
<td>26.46</td>
</tr>
<tr>
<td>5</td>
<td>0.0640</td>
<td>0.2530</td>
<td>25.30</td>
</tr>
<tr>
<td>6</td>
<td>0.0600</td>
<td>0.2449</td>
<td>24.49</td>
</tr>
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<td>7</td>
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<td>0.2390</td>
<td>23.90</td>
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<tr>
<td>8</td>
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<td>0.2345</td>
<td>23.45</td>
</tr>
<tr>
<td>9</td>
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<td>0.2309</td>
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</tr>
<tr>
<td>10</td>
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<td>0.2030</td>
<td>20.30</td>
</tr>
<tr>
<td>200</td>
<td>0.0406</td>
<td>0.2015</td>
<td>20.15</td>
</tr>
<tr>
<td>300</td>
<td>0.0404</td>
<td>0.2010</td>
<td>20.10</td>
</tr>
<tr>
<td>1,000</td>
<td>0.0401</td>
<td>0.2003</td>
<td>20.03</td>
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<tr>
<td>10,000</td>
<td>0.0400</td>
<td>0.2000</td>
<td>20.00</td>
</tr>
</tbody>
</table>

1. Portfolio variance when a 3 percent return \( r \) is expressed as \( r = 0.03 \).
2. Portfolio standard also when a 3 percent return is expressed as \( r = 0.03 \).
3. Portfolio standard when a 3 percent return is expressed as \( r = 3.0 \).
Can You Do Well While Doing Good?

Mark Hebner is the founder and President of Index Funds Advisors, Inc. and Investing for Catholics, author of ifa.com and indexfunds.com, and the book, *Index Funds: The 12-Step Program for Active Investors*. He has been nominated as the author of one of the three all-time greatest investment books, along with Warren Buffett and John Bogle. Mark’s mission is to change the way the world invests by replacing speculation with science. Mark’s passion and commitment have made him a favorite lecturer on the subject of investing, and he is also considered the leading internet provider of information on investing.

Mary Brunson is the Vice President of Investing for Catholics. A former financial writer/consultant to financial services providers, Mary learned firsthand the significant advantages of passive investing and shares the company’s high level of commitment to educate all investors on the Science of Investing. She embarked on the specific mission of Investing for Catholics to advance the financial educations of Catholic investors and institutions. Investing for Catholics was developed to counteract a pervasive lack of reliable information about prudent and risk-appropriate investing among Catholic investors, and an absence of passively managed index investments that adhere to the values that are in keeping with the Catholic faith. Mary is also a member of the Marketing and Media Board at the Magis Center of Reason and Faith where she works closely with Fr. Robert J. Spitzer, S.J., Ph.D.

Harry Markowitz is an Academic Consultant to Investing for Catholics. Professor Markowitz is also an economist at the Rady School of Management at the University of California, San Diego. He is best known for his groundbreaking analysis of the effects of asset risk, correlation and diversification on expected investment portfolio returns. He is widely regarded as the “Father of Modern Portfolio Theory,” the movement which he sparked and earned him the 1990 Nobel Prize in Economics. In 1952, Professor Markowitz developed the profound notion that investors must consider not only return, but the risk associated with their investments. His research on portfolio selection and construction serves as the framework for the Prudent Investor Rule, with an estimated $7 trillion of institutional assets invested according to his principles. Dr. Markowitz is also the recipient of the 1989 John von Neumann Theory Prize.