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## **Luck, Skill and Investment Performance**

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# **Luck, Skill and Investment Performance**

## **Abstract**

This article presents a simple procedure for assessing the relative impact of luck and skill in determining investment performance. The procedure is then applied to the large cap value managers. The results are consistent with earlier work that suggests that the great majority of the cross-sectional variation in fund performance is due to random noise.

## **1. Introduction: The basic problem of skill versus luck**

Successful investing, like most activities in life, is based on a combination of skill and serendipity. Distinguishing between the two is critical for forward looking decision making because skill is relatively permanent while serendipity, or luck, by definition is not. An investment manager who is skillful this year presumably will be skillful next year. An investment manager who was lucky this year is no more likely to be lucky next year than any other manager.

The problem is that skill and luck are not independently observable. Instead all that can be observed is their combined impact which is here called performance. The central question, therefore, is to determine how much can be learned about skill by observing performance. It turns out that there is a straightforward way to investigate that question based on application of the bivariate normal distribution. Though the results presented here are well known in statistics, they are not commonly applied in the context of assessing portfolio managers. As shown, they can serve as the basis of a simple and useful model for assessing the skill of competing fund managers. To illustrate how the model works, I use the procedure to analyze the performance of large cap equity managers tracked by Morningstar. It turns out, as one might expect given the volatility of asset prices, that the relative performance of managers in any given year provides little information about management skill.

## **2. A simple model for assessing luck and skill**

To develop the model, assume that there exists a measure of performance,  $p$ , that reflects the sum of skill,  $s$ , and luck,  $L$ . This formulation has a straightforward

interpretation in terms of portfolio management. In that context,  $p$  represents the observed return on a specific portfolio,  $s$  represents the added expected return due to the skill of the investment manager, and  $L$  represents the impact of idiosyncratic risk on the portfolio's return over the observed holding period.

More specifically, assume that both luck and skill are normally distributed in the cross section and that

$$p = s + L , \tag{1}$$

where  $s \sim n(E(s), sd(s))$  and  $L \sim n(0, sd(L))$ . By definition, the mean of the luck distribution is zero. Because  $p = L + s$ ,  $p$  and  $s$  are distributed as bivariate normal with mean vector  $[E(s), 0]$  and covariance matrix

$$\begin{matrix} sd(p) & corr(p,s)*sd(p)*sd(s) \\ corr(p,s)*sd(p)*sd(s) & sd(s) \end{matrix}$$

Because luck cannot be correlated with skill, otherwise there would be a predictable component of luck, it follows that

$$sd(p) = sd(s) + sd(L) \text{ and } corr(p,s) = sd(s)/sd(p) . \tag{2}$$

For the bivariate normal distribution, it is well known from the statistical literature<sup>1</sup> that

$$E(s|p) = E(s) + corr(p,s)*sd(s)/sd(p)*[p - E(p)] . \tag{3}$$

Substituting the relations from (2) into (3) gives,

$$E(s|p) = E(s) + [var(s)/var(p)]*[p-E(p)] . \tag{4}$$

Because  $E(L) = 0$ , it follows that  $E(p) = E(s)$  so equation (4) can be written,

$$E(s|p) = E(s) + [var(s)/var(p)]*[p-E(s)] . \tag{5}$$

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<sup>1</sup> See Mood (1974).

Equation (5) is the basic model. It states that when performance is observed in excess of the mean, the assessment of skill is adjusted upward, but not all the way to the observed level of performance,  $p$ . Instead, the assessment of  $s$  is adjusted upward by the observed superior performance,  $p - E(s)$ , times the ratio of the variance of  $s$  to the variance of  $p$ . Therefore, the assessment of skill based on the observation of performance depends critically on  $\text{var}(s)/\text{var}(p)$ .

Notice that in both limiting cases, equation (5) makes intuitive sense. If  $\text{var}(L)$  is much larger than  $\text{var}(s)$ , then  $\text{var}(p) \gg \text{var}(s)$  in which case  $E(s|p)$  goes to  $E(s)$ . That is reasonable because if performance is dominated by luck, then observation of performance should play little role in the assessment of skill. On the other hand, if  $\text{var}(s) \gg \text{var}(L)$  then  $\text{var}(s)$  is approximately equal to  $\text{var}(p)$ , which implies  $E(s|p)$  goes to  $p$ . That makes sense because if luck has a relatively minor impact on performance, then observed performance is a precise measure of skill.

Equation (5) has numerous applications in finance and is the basis for the phenomenon referred to as regression toward the mean. Regression toward the mean occurs because whenever the measure of performance,  $p$ , differs from the mean that indicates two things. First, it indicates that the above average performance represents above average skill. Second, it indicates that the above average performance reflects good luck. In other words, above average performance is evidence of *both* good luck and superior skill. However, whereas the skill element is permanent, the luck element is transitory. Therefore, the expected performance next period reverts back toward the mean from  $p$  because the luck variable has an expected value of zero. The greater  $\text{var}(L)$  relative to  $\text{var}(s)$  the larger the regression back toward the mean.

What is here called luck can be interpreted as random measurement error in other contexts. Consider, for instance, the problem of estimating beta. In that case, skill is equivalent to the unobservable true beta and performance is the estimate of beta. For betas, we know that the  $E(s)$  is 1.0. Therefore, equation (5) says that the best estimate of beta is not the observed regression coefficient (measured by  $p$ ), but  $E(s|p)$  which is a weighted average of the estimated regression coefficient and the overall mean of 1.0. Based on this property, Merrill Lynch developed a widely adopted weighted average procedure for estimating beta. The task here, however, is not to estimate beta, but to assess the contributions of luck and skill in determining investment performance. The next section considers the application of equation (5) in that context.

### **3. Application of the model to mutual fund data**

To illustrate the evaluation procedure, it is applied here to data on mutual fund performance. Before turning to the data there is one important caveat. The model attributes investment performance exclusively to the combination of skill and luck. When the performance measure,  $p$ , is interpreted as the return on a portfolio, a third element comes into play namely the risk level of the portfolio. There are two ways to account for this. One is to perform the calculations in terms of risk adjusted returns, but that introduces the problem of deciding how to adjust for risk, a problem that the finance profession has not fully resolved after 40 years of research. The other approach is to perform the analysis on a comparable cohort of investment funds. That is the approach taken here.

The data are drawn from a comprehensive 2004 Morningstar database of mutual fund performance.<sup>2</sup> To assure that the data are relatively homogenous the sample is limited to funds that invest primarily in the U.S. large capitalization value stocks. The Morningstar database includes performance data on 1,034 large cap value funds during 2004. The first line of Table 1 presents cross-sectional summary statistics for the returns on these funds which serves as the measure of performance,  $p$ . As shown in the table, the mean return for the 2004 is 25.02% and the standard deviation across the 1,034 funds is 5.47%.

To apply equation (5) it is also necessary to estimate the standard deviation of  $s$ . This is more difficult because  $s$  is not directly observable. There are two distinct approaches for overcoming this difficulty. The first is to rely on judgment rather than specific data. For example, an investor may conclude that the stock market is sufficiently competitive that differences in skill among large cap value managers should lead to no more than a 200 basis points differential from the mean for the vast majority of funds. That judgment translates into a standard deviation of  $s$  on the order of 1.0%. If the standard deviation of  $s$  is taken to be 1.0%, then the ratio of  $\text{var}(s)$  to  $\text{var}(p)$  of 0.033. That ratio implies that the observation of annual performance should have virtually no impact on assessment of the relative skills of the 1,034 large cap value managers. This result is largely consistent with a large body of literature on mutual fund performance beginning with the classic work of Jensen [1968] and continuing up through the work of Nitzsche, Cuthbertson and O'Sullivan [2007].

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<sup>2</sup> The Morningstar historical data were graciously provided by Wilshire Associates.

An alternative approach is to use long-run return data to estimate the standard deviation of  $s$ . If the fund return data were stationary and the sample period were sufficiently long, then the cross-sectional standard deviation of  $s$  could be estimated with little error. Unfortunately, neither is the case. With a maximum sample period of fifteen years, luck still places a role in determining the cross-sectional distribution of returns. This random noise results in an upward biased estimate of the standard deviation of  $s$ . On the other hand, the 15-year sample is also impacted by survival bias. Whereas there are annual data for 1,034 funds, the 15-year sample contains only 341 funds. Because the funds that disappear from the sample are more likely to be underperformers, both the mean 15-year return and the cross-sectional standard deviation are likely to be overstated. Given that the calculation presented here is only illustrative, no attempt is made to adjust for either of these offsetting effects on the estimated standard deviation of  $s$ .

The second line of Table 1 presents the cross-sectional summary statistics for the 341 funds in the 15-year sample. The mean annual return 10.03% and the standard deviation is 1.57%. A standard deviation of 1.57% for  $s$  implies that the ratio of  $\text{var}(s)$  to  $\text{var}(p)$  is 0.082. This indicates that approximately 92% of the cross-sectional variation in annual performance is attributable to random chance.

### **3. Conclusion**

The simple model presented here provides a useful, practical tool for assessing the impact of skill and luck on portfolio performance. When the model is applied to a sample of large cap value managers, the results indicate the most of the annual variation in performance is due to luck, not skill. This finding is consistent with that reported in

other papers on mutual fund performance. Nonetheless, the model provides another way of analyzing performance data.

The analysis also provides further support for the view that annual rankings of fund performance provide almost no information regarding management skill. Potential investors are better advised to consider the stated investment philosophies of competing firms than to rely on such rankings. In any event, at best minors revisions of estimates of skill such be based on annual performance data.

## REFERENCES

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Table 1

Cross-sectional Statistics for Large Cap Value Funds

Annual data for period ended March 2004\*

Number of funds	Mean return	Standard deviation
1,034	25.35%	5.47%

Fifteen-year data for period ended March 2004\*

Number of funds	Mean return	Standard deviation
341	10.03%	1.57%

\* All data from Morningstar